9-4/9-5 Interval of Convergence

Learning Objec□ves:

I can determine the radius and interval of convergence of a Taylor (or MacLaurin) series

I can determine the convergence at an endpoint of an interval of convergence for a Taylor or(or MacLaurin) series.

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If the power series is geometric, it is easy to find the interval of convergence and you can find the actual sum of the infinite series.

Remember,

$$\sum_{n=1}^{\infty} x^n = 1 + x + x^2 + x^3 + x^4 + \ldots \qquad a_1 = 1 \quad r = x$$

is a geometric series with a_1=1 and r = x.

$$S = \frac{a_1}{1-r} \text{ so } 1 + x + x^2 + x^3 + x^4 + \ldots = \frac{1}{1-x}$$

and will converge whenever |r|<1. $|x| < 1$ $\quad -1 < x < 1$

When the series is NOT geometric, things are a li□le more complicated and we CANNOT find the sum of the infinite series.

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Ex1. Find the interval of convergence for each Power Series

1. $f(x) = \dfrac{5x}{1-9x}$

2. $f(x) = \dfrac{3x^3}{1-x^2}$

3. $f(x) = \dfrac{7}{1-(3x+5)}$

4. $f(x) = \dfrac{-2x}{1+4x^2}$

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Convergence Theorem for Power Series

There are 3 possibili□es for $\sum_{n=0}^{\infty} c_n (x-a)^n$ with respect to convergence:

1.) There is a # R such that the series diverges for |x-a|>R but converges for |x-a|<R. This series may or may not converge at either endpoint

2.) The series converges for all x ($R = \infty$)

3.) The series converges for x=a and diverges everywhere else (R=0)

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Ex2. Show that the McLauren Series for f(x)=sin(x) converges to sin(x) for all x.

$$\sin x \Rightarrow \overset{n=0}{x^1} - \overset{n=1}{\frac{x^3}{3!}} + \overset{n=2}{\frac{x^5}{5!}} - \overset{n=3}{\frac{x^7}{7!}} + \cdots$$

$$\sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{(2n+1)!}$$

RATIO TEST

$$\lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right| < 1$$

$$\lim_{n\to\infty} \left| \frac{\dfrac{x^{2(n+1)+1}}{[2(n+1)+1]!}}{\dfrac{x^{2n+1}}{(2n+1)!}} \right| \overset{?}{<} 1$$

$$\lim_{n\to\infty} \left| \frac{x^{2n+3}}{(2n+3)!} \cdot \frac{(2n+1)!}{x^{2n+1}} \right| < 1$$

$$\lim_{n\to\infty} \left| \frac{\cancel{x^{2n}} \cdot \cancel{x^{3}}\,x^2}{(2n+3)(2n+2)\cancel{(2n+1)!}} \cdot \frac{\cancel{(2n+1)!}}{\cancel{x^{2n}} \cdot \cancel{x^1}} \right| < 1$$

$$\lim_{n\to\infty} \left| \frac{x^2}{(2n+3)(2n+2)} \right| < 1$$

$$|x^2| \cdot \lim_{n\to\infty} \left| \frac{1}{(2n+3)(2n+2)} \right| < 1$$

$$x^2 \cdot \lim_{n\to\infty} \left| \frac{1}{(2n+3)(2n+2)} \right| < 1$$

$$x^2 \cdot 0 < 1$$

$$-\infty < x < \infty$$

x converges for $\mathbb{R}$

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Ex3. Show that the interval of convergence for the Taylor Series f(x)=ln(x) centered at x=1 is (0, 2].

$$\ln x \Rightarrow \overset{n=0}{(x-1)} - \overset{n=1}{\tfrac{1}{2}(x-1)^2} + \overset{n=2}{\tfrac{1}{3}(x-1)^3} - \overset{n=3}{\tfrac{1}{4}(x-1)^4} + \ldots$$

$$\sum_{n=0}^{\infty} (-1)^n \frac{(x-1)^{n+1}}{n+1}$$

RATIO TEST

$$\lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right| < 1$$

$$\lim_{n\to\infty} \left| \frac{\dfrac{(x-1)^{n+2}}{n+2}}{\dfrac{(x-1)^{n+1}}{n+1}} \right| \overset{?}{<} 1$$

$$\lim_{n\to\infty} \left| \frac{(x-1)^{n+2}}{n+2} \cdot \frac{n+1}{(x-1)^{n+1}} \right| < 1$$

$$\lim_{n\to\infty} \left| \frac{\cancel{(x-1)^n}(x-1)^2}{n+2} \cdot \frac{n+1}{\cancel{(x-1)^n}\cancel{(x-1)}} \right| <$$

$$\lim_{n\to\infty} \left| (x-1) \cdot \frac{n+1}{n+2} \right| < 1$$

$$|x-1| \lim_{n\to\infty} \left| \frac{n+1}{n+2} \right| < 1$$

$$|x-1| \cdot 1 < 1$$

$$|x-1| < 1$$

$$-1 < x-1 < 1$$

$$0 < x < 2$$

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Testing the Endpts

$$\sum_{n=0}^{\infty} (-1)^n \frac{(x-1)^{n+1}}{n+1}$$

$x=0$

$$\sum_{n=0}^{\infty} (-1)^n \frac{(0-1)^{n+1}}{n+1}$$

$$= \sum_{n=0}^{\infty} \frac{(-1)^n (-1)^{n+1}}{n+1}$$

$$= \sum_{n=0}^{\infty} \frac{(-1)^{2n+1}}{n+1}$$

$$= -1 + \frac{-1}{2} + \frac{-1}{3} + \frac{-1}{4} + \cdots$$

Non-alt Harmonic

Diverge @ $x=0$

$x=2$

$$\sum_{n=0}^{\infty} (-1)^n \frac{(2-1)^{n+1}}{n+1}$$

$$= \sum_{n=0}^{\infty} (-1)^n \frac{(1)^{n+1}}{n+1}$$

$$= \sum_{n=0}^{\infty} (-1)^n \frac{1}{n+1}$$

$$= 1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \cdots$$

Alt. Harmonic

$\lim_{n\to\infty} \frac{1}{n+1} = 0$

converges @ $x=2$

$$0 < x \le 2$$

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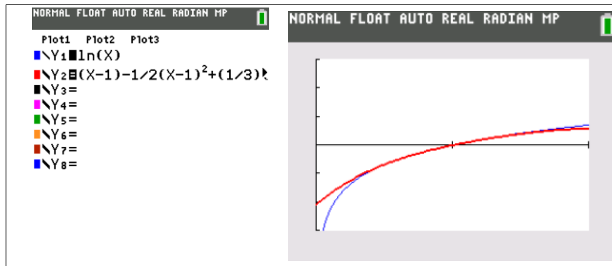

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Ex3. Find the interval of convergence for each power series

a.) $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{x^{2n}}{2n}$

b.) $\sum_{n=0}^{\infty} \frac{(10x)^n}{n!}$

c.) $\sum_{n=0}^{\infty} n!(x+1)^n$

d.) $\sum_{n=1}^{\infty} \frac{(x-3)^n}{2n}$

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Homework

Pg 511 # 8, 9, 11, 14, 16, 17, 18, 49, 52, 55-60

Pg 523 #55, 56, 66, 68, 69, 70

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